ME280A - Introduction to the Finite Element Method

COURSE OUTLINE

I. Introduction to the course; generalities (1)

1. Historical perspective: the origins of the finite element method.
2. Introductory remarks on the concept of discretization.
3. Classifications of partial differential equations.

II. Mathematical preliminaries (3)

1. Linear function spaces, operators and functionals.
2. Continuity and differentiability.
3. Inner products, norms and completeness.
4. Background on variational calculus.

III. Methods of weighted residuals (4)

1. Galerkin methods.
2. Collocation methods.
3. Least-squares methods.

IV. Variational methods of approximation (2)

1. Rayleigh-Ritz method.
2. Variational theorems.

V. Construction of finite element subspaces (5)

1. Compatibility and completeness of admissible spaces.
2. Basic element shapes in one, two and three dimensions.
3. Polynomial shape functions.
4. Area coordinates.
5. The concept of isoparametric mapping.

VI. Computer implementation of finite element methods (3)

1. Numerical integration of element matrices.
2. Computer program organization.
3. Assembly of global finite element arrays.
4. Algebraic equation solving by Gaussian elimination and its variants.
5. Finite element modeling: mesh design and generation.

VII. Elliptic differential equations (4)

1. Application to linear elasticity.
2. The patch test.
3. Non-conforming finite element methods.
4. Best approximation property of the finite element method.
5. Some basic error estimates.
6. Application to Stokes flow: treatment of constraints via the Lagrange multiplier technique and its regularizations: a prelude to mixed methods.

VIII. Parabolic differential equations (3)

1. Application to the transient heat equation.
2. Standard semi-discretization methods.
3. Stability and accuracy of classical time integrators.
4. Discontinuous Galerkin and space/time finite element methods.

IX. Hyperbolic differential equations (3)

1. Application to linear elastodynamics: Newmark integrators and the θ-method.
2. Numerical analysis of the Newmark family of integrators: stability and accuracy properties.
3. Application to convection-diffusion problems: standard Galerkin method, classical artificial diffusion and streamline diffusion methods.