

ME281 - Methods of Tensor Calculus and Differential Geometry

COURSE OUTLINE

I. Introduction

1. Historical perspective.
2. Euclidean and Riemannian geometry.
3. Classical view of tensors.

II. Modern view of tensors

1. Background on set theory and topology.
2. Vector spaces and duality.
3. Covariant and contravariant tensors.

III. Vector and tensor analysis on smooth manifolds

1. The notion of a smooth manifold.
2. Vectors and tangent space at a point.
3. Differential of a real-valued function.
4. One-forms and cotangent space at a point.
5. Tensors on manifolds.
6. The special case of Riemannian manifolds.
7. Push-forward and pull-back of vectors and one-forms.
8. Flow of vector fields.
9. Lie derivative of vectors and one-forms.
10. Wedge product and exterior/interior differentiation.

IV. Riemannian geometry

1. Connections and covariant differentiation on a manifold.
2. Parallel translation of vectors on a curve.
3. The fundamental theorem of Riemannian geometry.
4. Curvature tensor and “flatness” of manifolds.

5. Geodesics.

V. Surfaces embedded in the Euclidean 3-space

1. The first and second fundamental forms.
2. The equations of Gauss and Mainardi-Codazzi.
3. Measures of curvature.
4. Gauss' *Theorema Egregium*.

VI. Lie groups

1. Definitions and examples.
2. Invariant vector and covector fields.
3. Lie algebra of a Lie group.
4. One-parameter subgroups and the exponential map.